Why we teach about mathematical reasoning in primary school

Children Learning Research Group
Department of Education
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What do numbers mean?

• Numbers are signs in a system
  – the signs represent something (quantities and relations)
  – the system is organised in such a way that it allows us to reach conclusions beyond the information we have

• Number systems have a logic
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We don’t memorise the complete number sequence. We understand the logic and generate numbers that we have not heard before.
The logic of number systems

• Additive composition
  – any number is composed of two other numbers

• The inverse relation between addition and subtraction
  – when you add and take away the same number of objects, the number does not change
  – if $6+3=9$, then $9-3=6$
Relations between quantities

• Additive relations
  – Rob and Anne have some books (quantity)
  – Anne has 5 books less than Rob (relation - or Rob has 5 more books than Anne)
  – Anne has 5 books. How many books do they have together?

• Multiplicative relations
  – Rob and Anne have together 15 books (quantity)
  – Rob has twice as many books as Anne (relation - or Anne has half the books that Rob has)
  – How many books does each one have?
In each house in this street live 3 dogs. How many dogs live in this street?

Reasoning comes before arithmetic.
Both additive and multiplicative reasoning can be used from the first year in school.
Mathematical reasoning uses different signs

– At first children are able to think about quantities by using signs that they can manipulate

– Children can also use drawings to represent quantities

– Restricting children’s solutions to the use of numbers can restrict their reasoning about quantities

– The programme plans a progression in the use of signs for reasoning
Using The Programme
General details

• The programme took place in 2013/2014.
• 17 schools were randomised to take part in the project; 19 formed the comparison group.
• The 17 schools had 24 classes and 24 teachers.
• A wide mix of schools participated from small rural schools to large city schools in areas of deprivation.
The structure of the programme

• There are 10 units of work which we expect will be taught over 12 weeks (more than one lesson should be available).
• There is a comprehensive teacher handbook which contains unit plans and detailed instructions of how the activities are run and the materials required.
• This information is also available on the website with the computer games.
The structure of the programme

• The class work together for two thirds of each unit.
• For the last third of the time, from Unit 2 onwards, the class splits into 2 groups.
• When they split into groups, one group plays online games which consolidate what has been taught in the lesson.
• The group not playing the computer games works with the teacher. This is often a pre-teaching time or an extension time for the more able children.
• The 2 groups alternate each week.
The best thing about the programme

• ‘It makes the teacher reflect and think about Maths.’
• ‘All of it! The underlying theory, the activities, how it is laid out, resources.’
• ‘Emphasis on inversion helps develop children’s understanding.’
• ‘Having to verbalise their thinking has really helped the children.’
Has the programme given you new ideas for teaching some things?

• ‘It has enabled me to use questioning in a more detailed way.’

• ‘Yes, particularly use of modelling, made me think about how children learn and how I teach, e.g. importance of children understanding inversion.’

• ‘Enjoyed teaching inversion and the challenge this provides for children.’
What are you finding challenging when delivering the programme?

• ‘Time management’
• ‘Fitting everything in’
• ‘Range of ability, correct pace for all children’
• ‘School resources – ‘computers not working’
• ‘Computers at opposite end of the school’
• ‘Computers not available in the mornings.’
• ‘Getting resources ready, printed etc. ‘
General Comments

• ‘I absolutely love it and the children are learning so much!’
• ‘Children love it’
• ‘The programme highlighted areas that we have passed on to other members of staff as areas to develop in the school.’

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